# COMMUNICATIONS TO THE EDITOR

## Agitation of Non-Newtonian Fluids

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This paper is a comment on several statements concerning the prior art in the paper, "Power Requirements and Blend Times in the Agitation of Pseudoplastic Fluids," by E. S. Godleski and J. C. Smith, which appeared on page 617 of the November, 1962, issue of the *A.I.Ch.E. Journal.*

The authors' introduction states that no prior art paper has considered power requirements in non-Newtonian fluids which deviate from the power-law (that is, fluids which possess a nonconstant flow behavior index), and two of their three conclusions center on their stated extension of prior-art correlations to include such materials. However, the fluids used by Metzner, Feehs, Lopez-Ramos, Otto, and Tuthill (reference *4* of the Godleski-Smith paper) were frequently not power law materials. The legends of Figures 2 and 6 show, for example, the ranges covered in these instances. Similar data for the other figures are available in the published theses to which reference is given in the paper by Metzner, et al. When it is noted that changes in flow behavior index become irrelevant in any event as the index approaches unity (the indirect method used to estimate mean shear rates in a mixer in both of these papers breaks down as the fluid approaches Newtonian behavior) and the actual changes given by Godleski and Smith are compared to those in the earlier work by Metzner, et al., one sees that the deviation from power law behavior was about the same in both instances although a greater variety of fluids was used in the earlier work. Thus, while this portion of the Godleski—Smith paper presents clearly useful results, these results appear to be of the nature of an independent verification of other work published earlier.

In the second portion of their paper, a discussion of mixing or blending times, comparisons are made to the earlier paper of Norwood and Metzner (reference *7* of the Godleski-Smith paper). Godleski and Smith state that the earlier work on Newtonian fluids predicted mixing or blending times which were ten to fifty times smaller than their measured values in non-Newtonian systems. While some difference might well be expected, these appear to be surprisingly large. Inspection of their complete data, kindly supplied by Professor Smith during our correspondence, do reveal appreciable differences but these are greatest near those conditions under which Norwood and Metzner suggest the occurrence of a sharp discontinuity in the non-Newtonian case. For example, at a $T/D$ ratio of approximately 3.0, Norwood and Metzner suggest that the non-Newtonian mixing times would approach infinity below a Reynolds number of about 270. This is just the behavior experienced by Godleski and Smith except that the critical Reynolds numbers range from approximately 500 to 1,000 instead of the value previously estimated. Unfortunately, no data appear to be available in either study to assess the differences or similarities between the mixing rates in Newtonian and non-Newtonian systems under conditions well removed from the transition point, below which the non-Newtonian mixing rates approach zero because of incomplete fluid turnover in the vessel.

The author does not wish to suggest strongly that the Norwood-Metzner correlation for mixing rates in Newtonian fluids be applied to non-Newtonians. As in the original paper by Metzner, et al. this can only be suggested as a rough and temporary expedient until some actual mixing-rate data are available for non-Newtonian systems. It is concluded, however, that under conditions of good mixing the differences may not be as large as in the transition region in which the Godleski-Smith data were obtained.

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## Reradiation to Furnace Tubes. Effect of Tube-to-Wall Clearance

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One of the most common radiant heat transfer problems in engineering calculations is the design of tubular industrial furnaces such as oil heaters, cracking furnaces, and steam boilers. Heat sinks in these furnaces are in the form of rows of tubes supported along the walls. A high temperature body of gas, either with or without suspended solid particles, constitutes the heat source.

Radiant fluxes to the sinks may be conceived of as being made up of two parts: direct radiation from the source and reradiation from the wall behind the tube rows.

The geometry of this radiating system is conveniently handled through conversion to a radiation system between two parallel planes. Hottel (*1*) evaluated factors of comparison between the real system and an idealized two-parallel-plane system. This factor of comparison, also called *effectiveness factor of tube rows* (*2*), was presented as a function of the ratio of tube-center spacing to tube diameter. Along with other cases, curves were given for a single row of tubes and double rows of staggered tubes. The results are widely used and contained in textbooks on heat transfer (*2, 3*).

# Abstracts and Key Words*

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### STATISTICS AND NUMERICAL METHODS IN CHEMICAL ENGINEERING

**Key Words:** A. Experiments-8, Design-8, Statistics-10, Sequencing-8, Costs-9, Economizing-9, Processes-1, Optimization-2, Mathematics-10, Response-, Surfaces-8, Scale-Up-2, Pilot Plant-5, Plant-5, Research-5, Planning-8, Efficiency-9, Effectiveness-9, Productivity-9, Savings-9. B. Information-8, Computers-10, Variables-9, Models-9, Regression-10, Equations-9, Parameters-9, Hypotheses- 9, Theories-9, Testing-10.

**Abstract:** An experimental program is described which modifies the accepted statistical design of experiment by introducing a sequential program for experimentation. This plan allows for more effective use of existing knowledge and for the direct application of new information gained during the course of the study. At each stage of the program, the current state of knowledge is assessed and then revised to converge most rapidly to the objectives of the project. Where existing knowledge is extensive, or new information is gained quickly, reduction in research, pilot plant, or plant costs may be considerable.

**Reference:** Harrington, Edwin C., Jr., Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 1 (1963).

**Key Words:** Random Numbers-1, Noise-1, Process Simulation-8, Process Optimization-8, Computer-10, LGP-30-10.

**Abstract:** Random numbers selected from arbitrary nonuniform distributions may be used to simulate, within a digital computer, those natural variables commonly met in engineering problems. This technique is applied to the simulation of a simple continuous chemical process and to the optimization of the operation of this process.

**Reference:** Petersen, D. R., Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 8 (1963).

**Key Words:** Fitting-5, Reaction Kinetics-1, Nonlinear Estimates-2, Coefficient Estimation-2, Reaction Data-1, Regression-4, Ill-Conditioned Data-3.

**Abstract:** This article discusses the problems, for the study of chemical reaction kinetics, in interpretation and computation associated with the procedure of evaluating alternative models and then statistically evaluating the results for goodness of fit. A particular set of experimental data are examined, and in the light of the results some tentative conclusions are drawn about the value of nonlinear model fitting. Two major conclusions are drawn: first, it is possible to distinguish between the goodness of fit of different kinetic models with fairly crude data and with simple curve fitting techniques; second, determination of the coefficients in the model requires a proper design of experiments and a somewhat more elaborate fitting procedure.

**Reference:** Blakemore, John W., and Arthur E. Hoerl, Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 14 (1963).

(*Continued on page 558*)

*For details on the use of these key words and the A.I.Ch.E. Information Retrieval Program, see Chem. Eng. Progr., 57, No. 5, p. 55 (May, 1961), No. 6, p. 73 (June, 1961); 58, No. 7, p. 9 (July, 1962).

It is important in the application of these results to realize the assumptions made in their derivation. Hottel assumed the radiating plane to be at infinite distance from the tubes. For all practical purposes, this assumption is satisfied by real systems. The contribution from direct radiation to the factor of comparison is therefore valid. He further assumed the reradiating wall to be at infinite distance from the tubes. This situation might not hold for actual systems.

It will be shown in the following discussion that tube-to-wall clearance makes considerable difference to reradiation received by the tubes. Reradiation increases as the clearance is increased. Thus, Hottel's curves present the limiting values.

Recently, Mathis, Schweppe, and Wimpress (*5*) and Sutherland (*4*) measured rate of heat transfer to tubes in fireboxes. Considerable increase in the rate was observed as the clearance between tubes and wall was increased. The observed variation was considered to be entirely owing to convection around the tubes. Results of the present analysis show that account should also be made of changes in reradiation in their experimental systems.

#### THEORETICAL DEVELOPMENT

The effect of tube-to-wall clearance on radiant flux is expressed through the factor of comparison, $\alpha$, of the tube row with a cold plane. The cold plane area, $A_{cp}$, is the product of three factors: tube center-to-center spacing, tube length, and number of tubes in a row. The effective absorptive area of a tube bank is expressed by the product $\alpha A_{cp}$.

Since $A_{cp}$ is independent of tube-to-wall clearance, variation in $\alpha$ with clearance will, therefore, directly express the relative variation of total flux received by the tube row.

The overall value of $\alpha$ is made up of two contributions: $\alpha_d$, from direct radiation and $\alpha_r$, reradiation from the wall. The analysis in this paper will be focused on $\alpha_r$ and $\alpha_d$ will be taken from Hottel's previous work (*1*).

The present analysis starts with Lambert's cosine law which states

$$dq_{2\to 1} = I_2 (\cos\theta_1 dA_1)(\cos\theta_2\, dA_2)/r^2 \qquad (1)$$

where

$dq_{2\to 1}$ = radiant flux from $dA_2$ to $dA_1$
$I_2$ = intensity of radiation from $dA_2$
$r$ = distance between $dA_1$ and $dA_2$
$\theta$ = angle between $r$ and the normal to the elemental area

When Equation (1) is applied to an area strip in the reradiating wall and

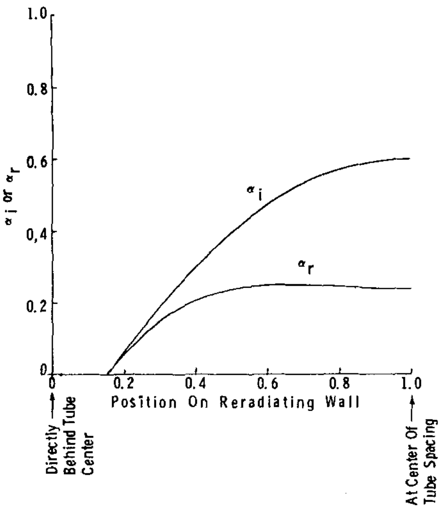


**Fig. 1. Point values of factor of comparison between two parallel planes. Tube center-to-center spacing = 2D; tube-to-wall clearance = 0; $\alpha_i$, for interception of radiation; $\alpha_r$, for reradiation.**

parallel to the center lines of the tubes, the relationship

$$dQ = I\ (\cos\phi\ dA)\,d\phi \qquad (2)$$

is derived. The derivation of Equation (2) from (1) is shown in the Appendix. $dA$ now stands for the elemental area strip. $\phi$ is the plane angle between the normal to $dA$ and the incident beam. $dQ$ is the flux impinging on $dA$ and bounded by $d\phi$.

The total flux impinging on $dA$ is then

$$Q = IdA\left[\int_o^{\phi_1} (\cos\phi\ d\phi) + \int_o^{\phi_2} (\cos\phi\ d\phi)\right] = IdA\ (\sin\phi_1 + \sin\phi_2) \qquad (3)$$

where $\phi_1$ and $\phi_2$, respectively, measure the angle of opening from the normal to either of the tangent lines. If the tubes were absent, both of these angles would be $\frac{\pi}{2}$ and the plane area $dA$ would receive a flux equal to

$$Q' = IdA\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = 2I\ dA \qquad (4)$$

The factor of comparison with two parallel planes in the interception of incident radiation is then

$$\alpha_i = \frac{Q}{Q'} = \frac{1}{2}\,(\sin\phi_1 + \sin\phi_2) \qquad (5)$$

The factor of comparison in reradiation is then

$$\alpha_r = \alpha_i\,(1 - \alpha_i) \qquad (6)$$

Equation (6) applies to the elemental area $dA$. For the reradiating wall as a whole, the factor of comparison is the area average of the point values.

The location of the elemental area might be such that more than two angles must be accounted for when the elemental area sees through more than one tube opening. This is readily carried out by the inclusion of additional terms, involving $\phi_3$, $\phi_4$, and so on in Equation (5).

The variation of $\alpha_i$ and $\alpha_r$ with position thus determined is illustrated in Figure 1. It can be seen that considerable deviation from uniformity exists at small values of tube-to-wall clearance. The term clearance here denotes the distance from the wall to the closest point on the tube surface.

Figure 2 shows average $\alpha_r$ as a function of tube center-to-center distance and tube-to-wall clearance. The case of infinite clearance is identical to Hottel's results.

The overall factor of comparison, $\alpha$, is then obtained by adding $\alpha_r$ to $\alpha_d$ where $\alpha_r$ is taken from Figure 2 and $\alpha_d$ is taken from Hottel's previous results for direct radiation to tube banks. Figure 3 shows the results.

## DISCUSSION

Reradiation to tube banks increases rapidly at small tube-to-wall clearance as the clearance is increased while keeping tube spacing constant. An asymptotic limit is achieved at a clearance approximately of the order of one tube diameter. There could be a variation of as much as 30% in reradiation from the case of zero clearance to the asymptotic value.

However, in terms of total radiant flux, the role played by tube-to-wall clearance is not as great, even though still possibly substantial. This is owing to the fact that most of the radiant flux comes as direct radiation which is independent of the tube-to-wall clearance.

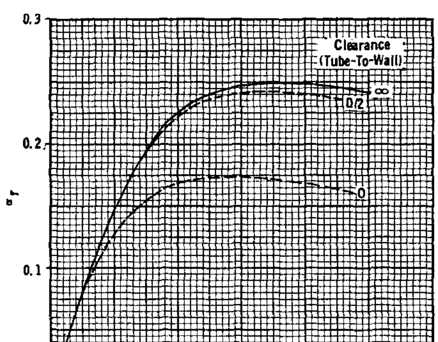


**Fig. 2. Factor of comparison of tube banks owing to reradiation. Single row of tubes against wall.**

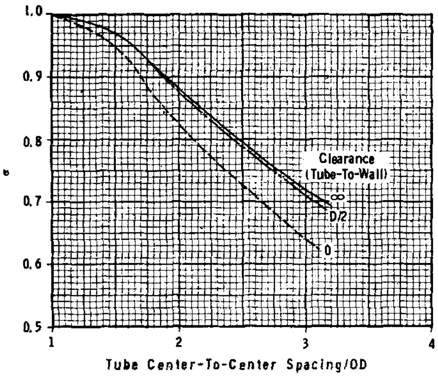


**Fig. 3. Effectiveness factor of tube banks owing to direct radiation and reradiation. Single row of tubes against wall.**

The results presented here are in qualitative agreement with experimental observations of previous investigators (*4, 5*) and should be useful to account properly for the reradiation contribution to their experimental systems.

Present practice in tube placement varies widely ranging from tubes partially embedded in the wall to tubes supported at a considerable distance from the wall. Reradiant flux to tubes placed against the wall has been frequently ignored. The present results show that this flux might be substantial and can be calculated.

## NOTATION

$A_{cp}$ = cold plane area
$dA$ = elemental area strip; plane area
$dQ$ = flux impinging on $dA$ and bounded by $d\phi$
$I_2$ = intensity of radiation from $dA_2$
$r$ = distance between $dA_1$ and $DA_2$

### Greek Letters

$\alpha$ = factor of comparison
$\alpha_i$ = interception
$\alpha_r$ = reradiation
$\alpha_d$ = direct radiation
$\theta$ = angle between $r$ and the normal to the elemental area
$\phi$ = plane angle between the normal to $dA$ and the incident beam

## LITERATURE CITED

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## ABSTRACTS AND KEY WORDS

Key Words: Statistics-10, Statistical Applications-10, Experimental Design-10, Quality Control-10, Sources of Variation-10, Factorial Designs-10, Analytical Variation-10, Production Variation-10, Analysis of Variance-10, Variance Components-10. Control Charts-10.

Abstract: A technique is presented for studying sources of variation in quality control data through a specially designed sampling and analytical program. The technique makes use of fractionated $2^n$ series factorial designs to schedule the work and novel factor control charts to point out assignable causes in each source of variation. Components of variation are calculated with analysis of variance.

Reference: Beazley, Charles C., Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 28 (1963).

Key Words: A. Optimization-8, Gas Scrubbing-10, Odor Acceptability-7, Household Product-1, Temperature-6, Gas/Liquid Ratio-6, Space Velocity-6, Packing Height-6. B. Designed Experiment-10, Paired Comparison-10, Statistical Analysis-1, Order Effect-8, Judges-4, Subjective Testing-10.

Abstract: The effect of gas scrubbing on the odor acceptability of an unpleasant smelling household product was studied in a designed experiment. The independent variables were column temperature, gas/liquid ratio, and packing height. Preference totals from a Scheffé paired comparison evaluation were used as responses. In the statistical analysis of these totals a large and significant order effect due to the desensitizing of the olfactory senses of the judges making the comparisons was confirmed.

Reference: Schneider, A. M., and A. L. Stockett, Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 34 (1963).

Key Words: A. Water Hardness-6, Water Temperature-6, Prior Use-6, Preference-7. B. Chi-Square-9, Probability-9, IBM-705-10.

Abstract: The consumer preference for a product is affected by environmental effects. Three conditions, temperature of water, hardness of water, and previous use of a product are studied for their independent and correlated effects on product preference. The statistical technique used is the $\chi^2$ analysis; most computations were made on the IBM-705. Some indications of interpretation problems are given.

Reference: Ries, P. N., and Harry Smith, Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 39 (1963).

Key Words: A. Kron-, Animated-8, Polyhedral-8, Models-8. B. Curve-9, Fitting-9, Engineering-9, Data-9. C. Mathematical-9, Model-9. D. Ewald-, Resonance-9, Theory-9, Crystals-9.

Abstract: The potential application of Kron's animated polyhedral models to the problem of curve fitting engineering data in several dimensions, when the underlying mathematical model is not fully known, is briefly summarized.

Reference: Dolby, James L., Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 44 (1963).

Key Words: Decision-Making-, Subjective Probability-, New Ventures-, Economic Evaluations-, Statistics-, Uncertainty-, Probability-, Judgment-, Estimates-, Experiments-, New Products-, Investment Decisions-.

Abstract: A nonmathematical exploration of the application of subjective probability and personal judgment in investment decisions where the unknowns are controlling, that is new product ventures, is presented.

Reference: Norton, John H., Chem. Eng. Progr. Symposium Ser. No. 42, 59, p. 49 (1963).

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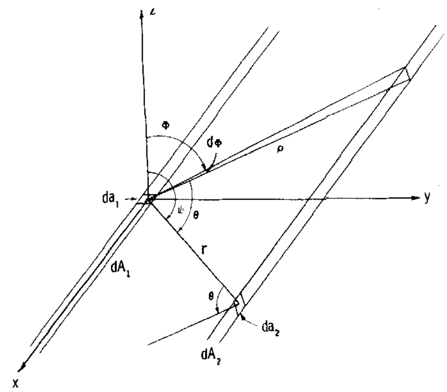


Fig. 4. Illustrating derivation of Equation (2).

5. Mathis, H. M., J. L. Schweppe, and R. N. Wimpress, *Petrol. Refiner*, 39, No. 4, p. 177 (1960).

## APPENDIX

### Derivation of Equation (2)

Consider the radiant flux from area strip $dA_2$ to area strip $dA_1$ as shown in Figure 4. $dA_1$ is on the wall and parallel to the center lines of the tubes. A coordinate system is chosen so that the $x$-axis coincides with the center line of $dA_1$. The origin is chosen to fall in the elemental area $da_1$ of $dA_1$. $dA_2$ is perpendicular to the $y$ $z$ plane. Furthermore, $dA_2$ is so oriented that its line of intersection with the $y$ $z$ plane is perpendicular to the line $\rho$ that connects this line segment to the origin.

The colatitude of $\rho$ is $\phi$. The colatitude of $r$ is $\psi$; and $r$ is the line connecting $da_1$ to $da_2$. The azimuth angle of $r$ from the $y$ $z$ plane is $\theta$.

Note that the usage of $\phi$ here is in agreement with that in Equation (2).

Apply Lambert's cosine law to $da_1$ and $da_2$

$$dq'_{2\rightarrow 1} = I_2 (\cos \psi da_1)(\cos \theta \, da_2)/r^2 \quad \text{(A1)}$$

Since

$$r = \rho/\cos\theta \quad \text{(A2)}$$

$$\cos\psi = \frac{\rho}{r}\cos\phi = \cos\theta\cos\phi \quad \text{(A3)}$$

$$da_2 = (\rho \, d\phi)\left(\frac{\rho \, d\theta}{\cos^2\theta}\right) \quad \text{(A4)}$$

substituting Equations (A2) to (A4) into (A1)

$$dq'_{2\rightarrow 1} = I_2 \, da_1 \cos^2\theta \cos\phi \, d\theta \, d\phi \quad \text{(A5)}$$

Integrating with respect to $\theta$ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, one obtains the total radiation from $dA_2$ to $da_1$

$$dq_{2\rightarrow 1} = \frac{\pi}{2} I_2 \cos\phi \, d\phi \, da_1 \quad \text{(A6)}$$

Integrating again with respect to $da_1$

$$dQ_{2\rightarrow 1} = \frac{\pi}{2} I_2 \cos\phi \, d\phi \, dA_1 \quad \text{(A7)}$$

Equation (A7) is identical to Equation (2), with $I = \frac{\pi}{2} I_2$.